Electromagnetic Questions

Question 1. A spatially dispersive material has $\mu = \mu_0$ and

$$\epsilon(\omega, \boldsymbol{k}) = \epsilon_0 + \frac{\chi_0}{-\omega^2 + i\lambda\omega + \omega_P^2 + k_z^2}$$

Calculate the dispersion relation for both Transverse $\boldsymbol{E}(t, \boldsymbol{x}) = E(t, z)\boldsymbol{i}$ and Longitudinal waves $\boldsymbol{E}(t, \boldsymbol{x}) = E(t, z)\boldsymbol{k}$ in the z-direction. I.e. the relationship between ω and $\boldsymbol{k}$ such that Maxwell's equations are solved by $\boldsymbol{E}(t, \boldsymbol{x}) = e^{i\omega t + \boldsymbol{k}\cdot\boldsymbol{x}}\boldsymbol{E}_0$

Question 2. A solenoid and a cylindrical magnet are designed to produce the same B field in a vacuum (As me to draw a picture).

If I place a rod of iron in the middle $\mu = 1000\mu_0$ describe the resulting B field near the rod.

Question 3. (a) Show that the two Maxwell's equations involving $\boldsymbol{D}$ and $\boldsymbol{H}$ are invariant under the "gauge" transformation

$$\boldsymbol{H} \to \boldsymbol{H} + \nabla\psi + \dot{\boldsymbol{\Psi}} \qquad \text{and} \qquad \boldsymbol{D} \to \boldsymbol{D} + \nabla \times \boldsymbol{\Psi}$$

(b) Are $\boldsymbol{D}$ and $\boldsymbol{H}$ measurable quantities.

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